UDC 539.3

CONTACT PROBLEM WITH ADHESION FOR AN ANISOTROPIC LAYER

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The mixed dynamic problem of the effect of two strip-like stamps on an orthotropic layer under plane strain conditions is considered. The stamps are assumed to adhere to the layer in the contact domain. The boundary value problem is reduced to a system of integral equations of the first kind, which is regularized by the factorization method from the side of the wide stamps. To solve the problem from the side of the narrow stamps, methods of singular integral equations of the "method of large λ" - type are used [1, 2], but in contrast to this method an approximate representation is proposed for the kernels which converges in the whole plane, this permits conjugation of the solutions obtained. Formulas describing the contact stresses under the stamps are obtained, and the nature of the stress singularity at the edge of the stamps is clarified. A numerical analysis is given of certain characteristics needed for construction of the solution.

1. Let us introduce a natural $Oxyz$ coordinate system, generated by the orthotropy of the layer, considering the z - axis perpendicular to its middle plane, and the axis of the stamps to form an angle φ with the y - axis, and a new $Ox'y'z'$ coordinate system by means of the relationships

$$x' + iy' = (x + iy)\, e^{i\varphi}, \quad z' = z$$

The problem posed is described in the $Ox'y'z'$ system by the Cauchy equations of motion and governing equations of the form [3]

$$\sigma_{11}' = c_{11}'\varepsilon_{11}' + c_{13}'\varepsilon_{33}', \quad \sigma_{33}' = c_{13}'\varepsilon_{11}' + c_{33}'\varepsilon_{33}', \quad \sigma_{13}' = 2c_{55}'\varepsilon_{13}'$$

Where $c_{ij}' = c_{ij}'(c_{ij}, \varphi)$, and $\varepsilon_{11}', \varepsilon_{33}', \varepsilon_{13}'$ and $\sigma_{11}', \sigma_{33}', \sigma_{13}'$ are, respectively, the strain and stress tensors in the $Ox'y'z'$ system. The boundary conditions of the problem have the form

$$z' = \pm h, \quad \sigma_{33}' = \sigma_{13}' = 0, \quad |x'| > a$$
$$u' = f_1^{\pm}(x')\, e^{-i\omega t}, \quad w' = f_2^{\pm}(x')\, e^{-i\omega t}, \quad |x'| \leqslant a$$

Here u', w' are components of the displacement vector in the new coordinate system, $f_i^{\pm}(x')$, $i = 1, 2$ characterizes the stamp shape, ω and $2a$ are the vibrations frequency and width of the stamps, and $2h$ is the layer thickness. Imposition of the radiation conditions, in whose derivation the limiting absorption principle is used [4], closes the formulation of the problem.

Let us study the steady-state vibrations regime and let us represent the displacement vector components in the form

$$u'(x', z', t) = u_0(x', z')\, e^{-i\omega t}, \quad w'(x', z', t) = w_0(x', z')\, e^{-i\omega t}$$

We separate the problem formulated into symmetric A and antisymmetric B in the coordinate z'. Later omitting the primes on the x', z', c_{ij}', ε_{ij}', σ_{ij}' and applying a Fourier transform in x, we reduce each of the problems A, B to a system of integral equations in the contact stresses of the following kind:

$$\int_{-a}^{a} \mathbf{k}(\xi - x)\, \mathbf{q}(\xi)\, d\xi = 2\pi \mathbf{f}(x), \quad |x| \leqslant a \tag{1.1}$$

$$\mathbf{k}(x) = \int_\sigma \mathbf{K}(\alpha)\, e^{i\alpha x} d\alpha, \quad \mathbf{K}(\alpha) = \begin{Vmatrix} K_{11}(\alpha) & iK_{12}(\alpha) \\ -iK_{12}(\alpha) & K_{22}(\alpha) \end{Vmatrix}$$

$$q(\xi) = \begin{Vmatrix} q_1(\xi) \\ q_2(\xi) \end{Vmatrix}, \quad f(x) = \begin{Vmatrix} f_1(x) \\ f_2(x) \end{Vmatrix}$$

After introduction of the dimensionless parameters

$$\varkappa^2 = \rho\omega^2 h^2 c_{33}^{-1}, \quad \gamma_1 = c_{11} c_{33}^{-1}, \quad \gamma_2 = c_{55} c_{33}^{-1}, \quad \gamma_3 = c_{13} c_{33}^{-1}$$

$$\gamma_4 = \gamma_3^2 + \gamma_2\gamma_3 - \gamma_1, \quad \alpha h = u$$

the elements of the matrix $\mathbf{K}(\alpha)$ have the following form in case B

$$K_{ij}(\alpha) = \delta A_{ij}(u), \quad i, j = 1, 2, \quad \delta = h c_{33}^{-1}(\gamma_3 + \gamma_2) \tag{1.2}$$

$$A_{11}(u) = -\mu_1\mu_2\,(\mu_1^2 - \mu_2^2)\,\mathrm{sh}\,\mu_1\,\mathrm{sh}\,\mu_2 D^{-1}(u)$$

$$A_{22}(u) = (\varkappa^2 - \gamma_1 u^2)(\mu_1^2 - \mu_2^2)\,\mathrm{ch}\,\mu_1\,\mathrm{ch}\,\mu_2\, D^{-1}(u)$$

$$A_{12}(u) = u\,[F(\mu_1, \mu_2) - F(\mu_2, \mu_1)]\, D^{-1}(u)$$

$$F(\mu_1, \mu_2) = \mu_1(\varkappa^2 - \gamma_1 u^2 - \gamma_3\mu_2^2)\,\mathrm{sh}\,\mu_1\,\mathrm{ch}\,\mu_2$$

$$D(u) = H(\mu_1, \mu_2) - H(\mu_2, \mu_1), \quad H(\mu_1, \mu_2) = F(\mu_1, \mu_2)\cdot (\varkappa^2 + \gamma_4 u^2 + \gamma_2\mu_1^2)$$

$$\{\gamma_2\mu^4 + [(1 + \gamma_2)\varkappa^2 + (\gamma_4 + \gamma_2\gamma_3)u^2]\mu^2 + (\varkappa^2 - \gamma_1 u^2)(\varkappa^2 - \gamma_2 u^2) = 0\}$$

$$\mathrm{Re}\,\mu_i \geqslant 0, \quad \mathrm{Im}\,\mu_i \geqslant 0, \quad i = 1, 2$$

where μ_1 and μ_2 are roots of the equation in the braces which satisfy the conditions mentioned.

The elements $A_{ij}(u)$ for the problem A are obtained from the formulas presented by replacing $\sinh\mu_k$ by $\cosh\mu_k$, and conversely.

The disposition of the contour σ in (1.1) governs the nature of the radiation at infinity, and involvement of an electronic computer is required to find it.

Curves of the zeros of $\det \mathbf{A}(u)$ of problem B are represented in Fig. 1, where curves of the poles $A_{ij}(u)$ are noted for the following values of the parameters by dashed lines [3]: $c_{11} = 11.66$, $c_{13} = 3{,}28$, $c_{33} = 11.04$, $c_{55} = 3.606$ (10^{11} dyne/cm^2), $\varphi = 0$.

Taking account of the shape of these curves, we can arrange the selection of the contour σ, which bypasses the positive poles from below, as a rule, and the negative poles from above. The elements $A_{ij}(u)$ satisfy all the conditions [5] except the asymptotic representation which in this case has the form

$$
\begin{aligned}
&A_{ii}(u) = C_i \,|\, u \,|^{-1} (1 + O(\,|\, u \,|^{-1})) \quad (i = 1,\ 2) \\
&A_{12}(u) = Bu^{-1} (1 + O(\,|\, u \,|^{-1})), \quad |\, u \,| \to \infty \\
&C_1 = dD^{-1}, \quad C_2 = \sqrt{\gamma_1}\, C_1, \quad B = (\sqrt{\gamma_1} - \gamma_3)\, D^{-1} \\
&d = [(\gamma_1 - \gamma_3^2)\ \gamma_2^{-1} + 2\, (\sqrt{\gamma_1} - \gamma_3)]^{1/2} \\
&D = (\gamma_2 + \gamma_3)\, (\gamma_1 - \gamma_3^2)
\end{aligned}
$$

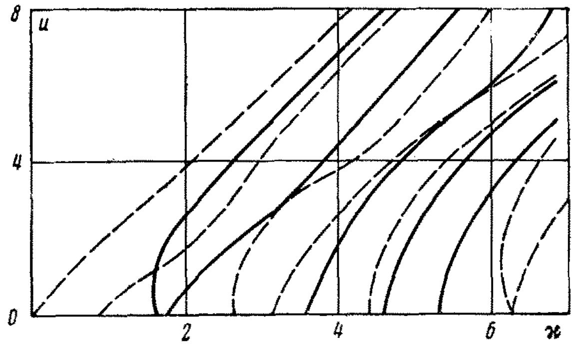

Fig. 1

To solve the system (1.1) from the side of the wide stamps, we apply the method of regularization based on factorization of the matrix functions [5] and resulting in the solution of a finite algebraic system whose order is determined to the accuracy given for the approximation of the integral operator

$$
\begin{aligned}
&\mathbf{X}(z_j) = \sum_{m=1}^{N} \mathbf{B}_{mj}^{(1)} [\mathbf{N}_+^{-1}(-z_m)\, \mathbf{Y}(z_m)\, e^{-iaz_m} + \mathbf{F}(-z_m)] \qquad (1.3) \\
&\mathbf{Y}(z_j) = \sum_{m=1}^{N} \mathbf{B}_{mj}^{(2)} [\mathbf{M}_-^{-1}(z_m)\, \mathbf{X}(z_m)\, e^{-iaz_m} + \mathbf{F}(z_m)], \quad j = 1, 2, \ldots, N \\
&\mathbf{A}(u) = \mathbf{N}_+^{-1}(u)\, \mathbf{N}_-(u) = \mathbf{M}_-^{-1}(u)\, \mathbf{M}_+(u) \\
&\Delta(u) = \det \mathbf{A}(u) = \Delta_+(u)\, \Delta_-(u) \\
&d_m^{(1)} = \frac{\Delta_-(-z_m)}{\Delta_-'(-z_m)}\, e^{-iaz_m}, \quad \mathbf{B}_{mj}^{(1)} = \mathbf{M}_-(-z_m)\, \frac{d_m^{(1)}}{z_m + z_j} \\
&d_m^{(2)} = \frac{\Delta_+(z_m)}{\Delta_+'(z_m)}\, e^{-iaz_m}, \quad \mathbf{B}_{mj}^{(2)} = \mathbf{N}_+(z_m)\, \frac{d_m^{(2)}}{z_m + z_j}
\end{aligned}
$$

Here $\mathbf{F}(u)$ is the Fourier transform of the right side in (1.1), z_m $(m = 1, 2, \ldots, N)$ are the zeroes of $\Delta(u)$ below the contour σ, and the factorization of the function $\Delta(u)$ and the matrix-function $\mathbf{A}(u)$ is performed with respect to the contour σ.

Using the solution (1.3), approximate formulas for the contact stresses under the stamps

$$
\delta \mathbf{q}(x) = \frac{1}{2\pi} \int_\sigma \mathbf{A}^{-1}(u)\, \mathbf{F}(u)\, e^{-iux} du + \qquad (1.4)
$$

$$+ i \sum_{m=1}^{N} [\mathbf{N}_{-2}^{-1}(-z_m)\,\mathbf{Y}(z_m)\,d_m^{(1)}e^{iz_m x} - \mathbf{M}_+^{-1}(z_m)\,\mathbf{X}(z_m)\,d_m^{(2)}e^{-iz_m x}], \quad |x| \leqslant a$$

can be obtained with an error $O(\exp(-2ba))$, $b > 0$, where the integral in the right side characterizes the degenerate component, and the sum is the influence of the stamp edges.

Therefore, realization of the factorization of the matrix $\mathbf{A}(u)$, which does not degenerate into a functionally commutative matrix in this case, is necessary for the construction of (1.4). A method to realize the approximate factorization of such matrices is proposed below.

Let us approximate the function (1.2) for fixed $\varkappa$ as follows:

$$A_{jj}^*(u) = C_j R_j(u)(u^2 + b^2)^{-1/2} \quad (j = 1, 2), \quad A_{12}^*(u) = Bu^{-1}R_0(u) \tag{1.5}$$

$$R_j(u) = \prod_{l=1}^{n_j}(u^2 - z_{jl}^2)\prod_{k=1}^{p}(u^2 - \alpha_{jk}^2)\prod_{i=1}^{m}(u^2 - \zeta_i^2)^{-1}(u^2 + A^2)^{-p}S_j(u)$$

$$S_j(u) = \begin{cases} 1, & n_j = m \\ \prod_{l=1}^{m-n_j}(u^2 + d_{jl}^2), & m > n_j \end{cases}, \qquad j = 0, 1, 2$$

Here z_{jl} and ζ_i $(j = 0, 1, 2)$ are the positive zeroes and poles of A_{12}, A_{11}, A_{22}, respectively; α_{ij}, A, d_{jl}, b are the approximation parameters, where the first two are associated with the application of Bernshtein polynomials and p is the degree of the polynomials.

Moreover, introducing the matrix

$$\mathbf{A}_1(u) = \mathbf{\Lambda}_-\mathbf{A}(u)\,\mathbf{\Lambda}_+, \quad \mathbf{\Lambda}_- = \begin{Vmatrix} C_2^{1/2} & 0 \\ 0 & C_1^{1/2} \end{Vmatrix}, \quad \mathbf{\Lambda}_+ = \begin{Vmatrix} C_1^{-1/2} & 0 \\ 0 & C_2^{-1/2} \end{Vmatrix}$$

we apply the process, described in [5], of approximate factorization of a matrix of special form to it by setting $C = \sqrt{C_1 C_2}$. Let us present the numerical results of constructing a right-sided factorization of the matrix $\mathbf{A}(u)$ for the problem B relative to the contour σ for $\varkappa = 1.0$, $b = 10$ with not more than 12% error

$$\mathbf{A}(u) = \mathbf{M}_-^{-1}(u)\,\mathbf{M}_+(u)$$

$$\mathbf{M}_-^{-1}(u) = \mathbf{\Lambda}_-^{-1}\mathbf{R}_-(u)\,\mathbf{G}_-(u)\,\mathbf{P}_-(u)\,Q_-(u), \quad \mathbf{M}_+(u) =$$

$$\mathbf{P}_+(u)\,\mathbf{G}_+(u)\,\mathbf{R}_+(u)\,\mathbf{\Lambda}_+^{-1}Q_+(u)$$

$$\mathbf{R}_\pm(u) = \frac{1}{2}\begin{Vmatrix} R_{11}^\pm(u) & iR_{12}^\pm(u) \\ -iR_{12}^\pm(u) & R_{11}^\pm(u) \end{Vmatrix}, \quad \begin{aligned} R_{11}^\pm(u) &= T_\pm(u) + T_\mp(-u) \\ R_{12}^\pm(u) &= T_\pm(u) - T_\mp(-u) \end{aligned}$$

$$T_\pm(u) = 1.892\frac{(u \pm 2.3i)(u \pm z^\pm)\dot{A}^4}{(u \pm 0.416)(u \pm 2.102)(u \pm iA)^4(b \mp iu)^{0\pm}}\prod_{k=1}^{4}(u - \alpha_k^\pm)$$

$$Q_\pm(u) = \frac{(u \pm 8.8i)(u \pm 9.8i)}{(u \pm z^+)(u \pm z^-)(u \pm 13.8i)^2}, \quad \mathbf{\Lambda}_+ = \begin{Vmatrix} 0.596 & 0 \\ 0 & 0.529 \end{Vmatrix},$$

$$\Lambda_- = \begin{Vmatrix} 1.892 & 0 \\ 0 & 1.866 \end{Vmatrix}$$

$$A = 4.39, \quad z^+ = 1.283, \quad z^- = 2.295, \quad \theta_\pm = 0.5 \pm 0.115i$$

$$\alpha_s^+ = \overline{\alpha_{21}^-}, \quad \alpha = \alpha^+\alpha^- = 0.016 + 0.138i$$

$$\mathbf{P}_\pm(u) = \begin{Vmatrix} \alpha^\pm & 0 \\ 0 & P_{22}^\pm(u) \end{Vmatrix}, \quad \mathbf{G}_+(u) = \begin{Vmatrix} 1 & \alpha^{-1}P_{12}(u) \\ 0 & 1 \end{Vmatrix}$$

$$\mathbf{G}_-(u) = \begin{Vmatrix} G_{11}(u) & G_{12}(u) \\ G_{21}(u) & G_{22}(u) \end{Vmatrix}, \quad P_{22}^+(u) = d_{22}^+ \prod_{\operatorname{Im}\beta_k<0} (u-\beta_k)$$

$$P_{22}^-(u) = d_{22}^- \prod_{\operatorname{Im}\beta_k>0} (u-\beta_k), \quad P_{12}(u) = \sum_{s=1}^{8} d_{12}^s u^{8-s}$$

$$G_{ij}(u) = \sum_{s=0}^{4} b_{ij}^s u^{4-s}, \quad d_{22}^+ d_{22}^- = 0.817 - 7.159i$$

$$b_{11}^0 = b_{12}^0 = b_{21}^0 = 0, \quad b_{22}^0 = -b_{21}^1$$

The coefficients b_{ij}^s, β_s, d_{12}^s, α_s^+ are presented in the Table 1

Table 1

	s	$z=b_{11}^s$	b_{12}^s	b_{21}^s	b_{22}^s	α_s^+		s	$z=\beta_s$	d_{12}^s	s	$z=\beta_s$	d_{12}^s
Re z	1	0	8.24	—0.82	2.44	—0.11	Re z	1	—0.04	8.24	5	0.04	73.78
	2	—8.24	23.00	—6.73	2.24	—0.05		2	—0.29	47.59	6	—3.20	19.15
	3	7.29	18.48	—3.98	0.53	0.44		3	—0.42	101.49	7	—1.02	1.82
	4	0.70	2.54	—0.40	0.00	—0.40		4	—0.34	133.25	8	—0.69	0.01
Im z	1	0	6.14	7.16	—21.34	—0.70	Im z	1	—0.03	6.14	5	0.04	36.11
	2	—6.14	16.12	—0.73	—19.60	—0.91		2	0.02	34.43	6	0.15	6.38
	3	—5.61	11.01	—0.38	—4.62	—1.09		3	0.15	69.83	7	—1.30	—0.33
	4	—3,82	—0.12	—0.02	0.00	—1.97		4	—0.26	75.22	8	1.23	—0.07

Qualitative investigations permit the conclusion that vibrations frequencies exist for $\varkappa = 2.70$, Fig. 1, for instance, such that not only the amplitudes and phases of each of the waves change for stamp rotation through a small angle $\Delta\varphi$ but their quantity can also vary (increase or decrease by one). This is related to the dependence of the curves shown in Fig. 1 on φ. The stresses at the stamp edges have a singularity of the form

$$q_k(x) \sim A(a \mp x)^{-1/2 \mp i\nu}, \quad x \to \pm a, \; k = 1, 2$$

$$\nu = \pi^{-1} \operatorname{arcth}(B / \sqrt{C_1 C_2})$$

We note that the method elucidated for the solution of the system (1.1) is effective for sufficiently wide stamps (for all a, in practice, because of selecting the parameter b large).

2. We shall use the method of singular integral equations for narrow stamps. Let us first clarify the structure of the kernel

$$k_{ij}^*(t) = \int A_{ij}^*(u)\, e^{iut} du$$

by using (1.5). For example, let us consider $k_{11}(t)$. Closing the contour σ in the upper half-plane for $t > 0$ as shown in Fig.2, and selecting that branch of the function $(z^2 + b^2)^{1/2}$ which takes on positive values as $z \to \infty$, we have

$$\int_{c_\sigma} \frac{R_1(u)\, e^{iut}}{\sqrt{u^2+b^2}} du = 2\pi i \sum_{k=1}^{m+1} \operatorname{res}\left(R_1(u)(u^2+b^2)^{-1/2} e^{iut}\right)\big|_{u=\zeta_k}, \quad \zeta_{m+1} = iA$$

Applying the Jordan lemma and expanding the function $R_1(u)$ into the simplest fractions, we consider one of the resulting integrals

$$I(t) = \int_{i\infty}^{ib} \frac{e^{iut} du}{\sqrt{u^2+b^2}\,(u^2-\zeta^2)} = \int_0^\infty (b^2 \operatorname{ch}^2 z + \zeta^2)^{-1} e^{-bt \operatorname{ch} z} dz \qquad (2.1)$$

$(u = ib \cosh z)$

It can be shown that $I(t)$ satisfies a differential equation with the initial conditions

$$I''(t) + \zeta^2 I(t) = K_0(bt) \qquad (2.2)$$

$$I(0) = \zeta^{-1} b^{-1} (1 + \lambda^2)^{-1/2} \ln(\lambda + \sqrt{1+\lambda^2}), \quad \lambda = \zeta / b$$

$$I'(0) = -\frac{\pi}{2}(b^2 + \zeta^2)^{-1/2}$$

$K_0(x)$ is the Macdonald function. The possibility of a passage to the limit in $I(t)$ and $I'(t)$ as $t \to 0$ and of differentiating under the integral with respect to the parameter $t \geqslant 0$ for $I(t)$ and $t \geqslant t_0 > 0$ for $I'(t)$ is shown by confirmations of the conditions of the appropriate theorem [6].

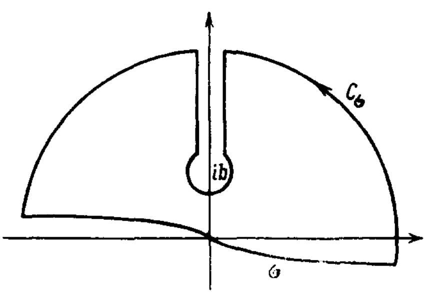

Fig.2

Knowing a series representation for $K_0(x)$ [7], it is natural to seek the solution of (2.2) in the form

$$I(t) = \sum_{k=0}^\infty a_k t^k \ln t + \sum_{k=0}^\infty d_k t^k$$

We obtain for a_k and d_k ($\psi(x)$ is the Euler psi-function)

$$a_{2n+2} = -\frac{1}{(2n+2)!} \sum_{k=0}^n (-1)^{n-k} \frac{(2k+1)!!}{(2k)!!} \frac{b^{2k} \zeta^{2(n-k)}}{2k+1},$$

$$a_{2n+1} = a_0 = 0$$

$$d_{2n+2} = \frac{1}{(2n+2)!} \sum_{k=0}^n (-1)^{n-k} \zeta^{2(n-k)} \Big\{-(4k+3)(2k)!\, a_{2k+2} +$$

$$b^{2k} \frac{(2k+1)!!}{(2k+1)(2k)!!} \left[-\ln\frac{b}{2} + \psi(k+1)\right]\Big\} + \frac{(-1)^{n+1}}{(2n+2)!} \zeta^{2n+2} d_0$$

$$d_{2n+1} = \frac{(-1)^n \zeta^{2n}}{(2n+1)!} d_1, \quad d_0 = I(0), \quad d_1 = I'(0), \quad n = 0, 1, 2, \ldots$$

On the basis of the theory of residues we have for $t > 0$

$$k_{12}^*(t) = iB\pi\left[2\sum_{k=1}^{m+1}\operatorname{res}(u^{-1}R_0(u)e^{iut})|_{u=\zeta_k} + 1\right], \quad \zeta_{m+1} = iA$$

Evaluating the residues and expanding the exponentials in series, we obtain the following representations of the kernels:

$$\begin{aligned} k_{jj}^*(t) &= 2C_j[-\ln|t| + F_{jj}(t)], \quad j = 1, 2 \\ k_{12}^*(t) &= iB\pi[\operatorname{sign} t + F_{12}(t)] \end{aligned} \tag{2.3}$$

the quantities $F_{ij}(t)$ are representable as uniformly convergent series in all planes

$$F_{jj}(t) = \sum_{s=0}^{\infty} f_{js}t^{2s+2}\ln|t| + \sum_{s=0}^{\infty} h_{js}|t|^s, \quad F_{12}(t) = \sum_{s=0}^{\infty} e_s t^{2s+1}$$

Inserting the representations (2.3) into the system (1.1), setting $x = ay$, $\xi = a\eta$, introducing new unknown functions by the relationship

$$\mathbf{q}(a\eta) = \mathbf{\Lambda}\mathbf{\Psi}(\eta), \quad \mathbf{\Psi}(\eta) = \left\|\begin{matrix}\Psi_1(\eta)\\ \Psi_2(\eta)\end{matrix}\right\|, \quad \mathbf{\Lambda} = \frac{1-i}{4}\left\|\begin{matrix}\varepsilon_2^{-1/2}, & i\varepsilon_2^{-1/2}\\ -i\varepsilon_1^{-1/2}, & -\varepsilon_1^{-1/2}\end{matrix}\right\|$$

and multiplying the system obtained on the left by $\mathbf{\Lambda}^{-1}$, we arrive at the system

$$\int_{-1}^{1}\mathbf{R}(a(y-\eta))\mathbf{\Psi}(\eta)\,d\eta = \pi\mathbf{g}(y) - \pi\int_{-1}^{1}\mathbf{S}(a(y-\eta))\mathbf{\Psi}(\eta)\,d\eta, \; |y| \leqslant 1 \tag{2.4}$$

$$\mathbf{R}(t) = \left\|\begin{matrix}R_+^*(t) & 0\\ 0 & R_-^*(t)\end{matrix}\right\|, \quad \mathbf{S}(t) = \left\|\begin{matrix}S_+(t) & iS_{12}(t)\\ -iS_{12}(t) & S_-(t)\end{matrix}\right\|$$

$$\mathbf{g}(y) = \left\|\begin{matrix}g_+(y)\\ g_-(y)\end{matrix}\right\|$$

$$R_\pm^*(t) = -\ln|t| \mp \frac{i\pi\varepsilon}{2}\operatorname{sign} t, \quad g_\pm(y) = \left[\sqrt{\varepsilon_2}\,\frac{1\pm i}{C_1}\times\right.$$
$$\left. f_1(ay) - \sqrt{\varepsilon_1}\,\frac{1\mp i}{C_2}f_2(ay)\right](\gamma_3+\gamma_2)^{-1}$$

$$S_\pm(t) = \frac{1}{2\pi}\left[F_{11}(t) + F_{22}(t) \mp i\pi\varepsilon F_{12}(t)\right], \quad S_{12}(t) =$$
$$\frac{1}{2\pi}[F_{11}(t) - F_{22}(t)]$$

$$\varepsilon = \sqrt{\varepsilon_1\varepsilon_2}, \quad \varepsilon_j = BC_j^{-1}, \quad j = 1, 2$$

Differentiating (2.4) with respect to y and regularizing the system obtained according to Vekua [8], we arrive at a system of Fredholm equations of the second kind. For small a this system can be solved by successive approximations [9].

Therefore, the whole domain of variation of the parameter a turns out to be covered by using the methods elucidated.

Notes. 1°. The results presented are carried over without difficulty to the general case of anisotropy.

2°. The above is valid even for static problems for which the need to construct the curves (Fig. 1) drops out and the form of (1.5) changes.

3°. At present, packets of applied programs have been developed for the solution of a system of the form (1.3) and the computation of contact stresses by means of (1.4).

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